BACKPAPER EXAMINATION

B. MATH III YEAR, II SEMESTER 2008-2009

INTRODUCTION TO STOCHASTIC PROCESSES

Answer as many questions as you can. Max. you can score is100. Time limit:

3 hrs

HMC stands for homogeneous Markov chain.

1. If $\{X_n\}$ is a HMC prove that

$P\{X_{n-1} = i_{n-1}|X_n = i_n, X_{n+1} = i_{n+1}\} = P\{X_{n-1} = i_{n-1}|X_n = i_n\}$ and find this conditional probabilty in terms of the transition matrix and the distribution of X_{n-1}. [10]

2. Let $\{X_n\}$ be a Galton-Watson process with offspring distribution $p_k = \frac{1}{2^{k+1}}(k \geq 0)$. Find $P\{X_2 = 5\}$.

Also find the extinction probability. [20]

3. Consider a two type Galton Watson process with offspring distribution $(\frac{1}{4}\frac{1}{2^{j+k}})$ for particles of type 1 and $(\frac{8}{15}\frac{1}{3^j 5^k})$ for particles of type 2. Prove that the probabilities of extinction are not both equal to 1.

Hint: Compute the mean matrix and its spectral radius. [20]

4. Consider a HMC with transition matrix $\begin{bmatrix} 0 & 0 & 1/2 & 1/2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$. Find all transient states, positive recurrent states and null-recurrent states. Find the period of each recurrent state. Find the mean return time for each recurrent state and find all stationary distributions. [25]

5. Let $\{X_t\}$ be a birth and death process with state space $\{0, 1, 2, ...\}$ and birth and death rates $\lambda_n = \lambda, \mu_n = n\mu(n \geq 0)$. Find the stationary distribution. You may assume that the equation $\sum_{i=0}^{\infty} \pi_i P_{ij}(t) = \pi_j (j \geq 0, t \geq 0)$ can be differentiated at $t = 0$ to get $\sum_{i=0}^{\infty} \pi_i q_{ij} = 0(j \geq 0)$. [25]

6. Let $\{X_t\}$ be a Piosson process with parameter λ. Prove that

$\lim_{s \to o} \frac{P\{X_{t+s} - X_t \geq n\}}{s^n}$ exists for each n and find the value of this limit. [10]